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Nonlinear vibration analysis of bladed disks with dry friction dampers

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Abstract

In this work, a new model is proposed for the vibration analysis of turbine blades with dry friction dampers. The aim of the study is to develop a multiblade model that is accurate and yet easy to be analyzed so that it can be used efficiently in the design of friction dampers. The suggested nonlinear model for a bladed disk assembly includes all the blades with blade to blade and/or blade to cover plate dry friction dampers. An important feature of the model is that both macro-slip and micro-slip models are used in representing dry friction dampers. The model is simple to be analyzed as it is the case in macro-slip model, and yet it includes the features of more realistic micro-slip model. The nonlinear multidegree-of-freedom (mdof) model of bladed disk system is analyzed in frequency domain by applying a quasi-linearization technique, which transforms the nonlinear differential equations into a set of nonlinear algebraic equations. The solution method employed reduces the computational effort drastically compared to time solution methods for nonlinear systems, which makes it possible to obtain a more realistic model by the inclusion of all blades around the disk, disk itself and all friction dampers since in general system parameters are not identical throughout the geometry. The validation of the method is demonstrated by comparing the results obtained in this study with those given in literature and also with results obtained by time domain analysis. In the case studies presented the effect of friction damper parameters on vibration characteristics of tuned and mistuned bladed disk systems is studied by using a 20 blade system. It is shown that the method presented can be used to find the optimum friction damper values in a bladed disk assembly.

1. Introduction

Forced vibration analysis of bladed disk assemblies is an important part of the design of gas turbine engines. One of the main problems of the gas turbine engines is the high cycle fatigue failure of turbine and compressor blades due to blade vibration resonance in the operating range. Preventing turbomachinery blade failures is an important issue for the gas turbine engine developers and therefore, avoiding excessive vibration in turbomachinery blading is crucial. Dry friction dampers dissipate energy in the form of heat due to the rubbing motion of the contacting surfaces resulting from relative motion. Therefore, placement of dry friction

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dampers between the blades and/or the blades and the cover plate is a widely used technique for decreasing vibration amplitudes.

Having dry friction in the system complicates the dynamic analysis of the system due to its nonlinear nature. In modeling friction dampers, two kinds of friction damping mechanisms have been used in literature. Macro-slip model, which assumes the entire friction surface as either slipping or stuck, is an extensively used method due to its mathematical simplicity [1–8]. Macro-slip models a friction damper as a spring one end of which slips if the force on the spring is greater than a certain value (Fig. 1a). In micro-slip model, which is more realistic, the entire friction surface is modeled as an elastic body, allowing local slipping in the friction element without gross slip [7–11]; therefore, it is possible to obtain damping even in the absence of gross slip. These results are also supported by experimental studies [11–13]; therefore, micro-slip model describes the friction behavior more accurately than the macro-slip model; however, it is mathematically complicated and computationally expensive.

Ferri and Dowell [14] studied the vibrations of a cantilever beam with dry friction damper placed at the end. The harmonic balance method with single and multiple harmonics was applied and experiments were conducted to verify the method. Ji et al. [15] analyzed a damped blade assembly under harmonic excitation. Normal force on the friction element was varied over a range in order to obtain an optimal normal load value for different excitation amplitudes. Ferri and Heck [16] studied dynamic behavior of a single-degree of freedom (dof) turbine blade with a friction damper, and developed three reduced order problems: small damper mass, zero damper mass and high damper stiffness by applying singular perturbation theory. Sanliturk et al. [8] analyzed a single blade using harmonic balance method and also performed an experiment on a similar system and compared the analytical and experimental results. Both macro-slip and micro-slip methods are used to define force–displacement characteristics of dry friction dampers in the analytical solution method. Menq et al. [9] developed a micro-slip model, in which the friction element has been modeled as an elastic bar that is in contact with a rigid surface and connected to a spring at the left end. Under the effect of uniform normal load distribution, partial-slip and gross-slip of the bar was analyzed and the force displacement curve for micro-slip was obtained.

Sanliturk et al. [17] developed a theoretical method to analyze underplatform dampers, in which harmonic balance method is employed and experimental hysteresis data are used in the solution procedure. Menq and Griffin [3] studied the forced response of frictionally damped beam and developed a method which uses finite element models for the analysis. An experiment was performed and results are compared with the ones obtained from the method developed. In another recent study, Sanliturk et al. [18] proposed a multidegree of freedom (mdof) model in order to simulate rainbow tests that are used to obtain optimum friction damper masses. In that work, authors used harmonic balance method in combination with a structural modification approach in order to perform simulations. A further review of friction dampers in turbo machinery applications can be found in Ref. [19].

In this study first a new friction model is developed, which is mathematically simple and computationally inexpensive as in the case of macro-slip, and accurate enough to represent the effects of micro-slip. Then it is used in a dynamic model of bladed disk assembly including multiblades and all friction dampers inserted

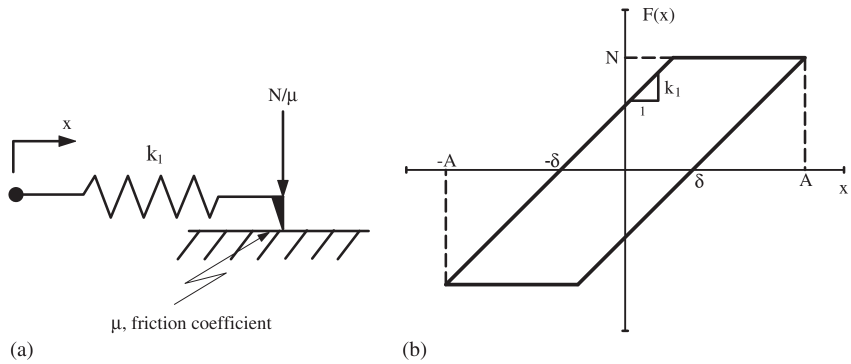

Fig. 1. (a) Schematic drawing, (b) corresponding hysteresis curve for macro-slip model.

between these blades or blades and cover plate. With the resulting nonlinear mdof model and a very efficient harmonic vibration analysis method, it is aimed in this study to introduce a model and a solution technique for bladed disk systems, which can be used for determining system response and thus optimal friction damper parameters.

Finite element models are developed and used for blade vibrations (for instance, see Refs. [20–22]), but nonlinearity introduced by friction dampers limits the dof of the model analyzed in case of mistuning. Since the cyclic symmetry of the system is destroyed, the whole bladed disk system should be considered instead of a single cyclic section, due to which system dof can increase dramatically. However, a very efficient harmonic vibration analysis approach developed for nonlinear mdof systems in previous studies [23–25] is employed in this work for the analysis of mdof model of bladed disk assemblies. The approach is based on expressing nonlinear force vector amplitude as a response-dependent matrix multiplied by the displacement amplitude. This has been achieved first by Budak and Özgüven [23] for certain type of nonlinearities and then by Tanrıkulu et al. [24] for a wide range of nonlinearities by using describing functions. In these work, the method developed by Özgüven [26] for the harmonic response analysis of non-proportionally damped linear structures has been extended to the harmonic vibration analysis of nonlinear structures. The basic approach in this method is to treat problem as a structural modification problem and determine harmonic response of a nonlinear system from that of the linear counterpart. Expressing nonlinearities in matrix form made it also possible to extend various other structural dynamic analysis methods developed for linear systems to nonlinear systems e.g., Refs. [17,18,27–29].

2. Vibration analysis of bladed disks

2.1. Lump parameter model

Bladed disk assemblies can be classified in three groups according to their blade configurations on the disks [30]. In shrouded bladed disks, blades are connected to each other, and according to the type of connection they are named as *tip-shrouded* or *mid-height shrouded*. If there is no connection between the blades, then this configuration is called *unshrouded*. In unshrouded and tip-shrouded bladed disk systems, each blade is modeled as a single-dof system, whereas in mid-height shrouded type each blade has two dof. As the disk is very stiff compared to the blades in a bladed disk system, in several studies, only the blades are considered in dynamic models. However, the lumped parameter model employed in this work in order to show the approach proposed, includes lumped model of the disk as well, which will help to capture some of the properties of the real system better. In such a discrete model, disk is modeled as an n-dof system, where n is the number of blades on the disk. A finite element model will definitely increase the accuracy of the results obtained, and it should be noted that the friction damper model developed and the solution method suggested in this study can easily be used with also a finite element model.

In this work, tip-shrouded bladed disks are used to explain the friction damper model and the analysis method developed (Fig. 2). In this model, M_i and m_i stand for the ith disk and ith blade masses, respectively. k_{ri}, k_{di}, and k_{si} correspond to the root stiffness between the ith disk element and ith blade, disk stiffness between the ith and $(i+1)$st disk elements, and shroud stiffness between the ith and $(i+1)$st blades, respectively.

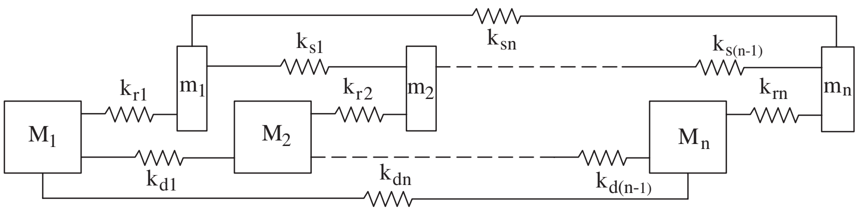

Fig. 2. Lumped parameter model for tip-shrouded bladed disk.

The equations of motion for the above described model of a tip-shrouded bladed disk system without friction dampers can be written in matrix form as

$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{C} \cdot \dot{\mathbf{x}} + i\mathbf{H} \cdot \mathbf{x} + \mathbf{K} \cdot \mathbf{x} = \mathbf{f}, \tag{1}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{H}$ and $\mathbf{K}$ represent the mass, viscous damping, structural damping, and stiffness matrices of the linear system, respectively, and i is the unit imaginary number. Here, $\mathbf{x}$ is the vector of displacements and dot denotes differentiation with respect to time. $\mathbf{f}$ represents the external harmonic forcing vector. In this equation the order of the matrices is $2n$, n being the number of blades. The ith element of the displacement vector corresponds to the displacement of the ith blade for $1 \leqslant i \leqslant n$, and displacement of the $(i-n)$th disk element for $n+1 \leqslant i \leqslant 2n$.

The existence of friction dampers in the system will make the system equations nonlinear. Without going into details, it can be said that due to the friction dampers, there will be an additional internal nonlinear force in system equations:

$$\mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{C} \cdot \dot{\mathbf{x}} + i\mathbf{H} \cdot \mathbf{x} + \mathbf{K} \cdot \mathbf{x} + \mathbf{N} = \mathbf{f}, \tag{2}$$

where $\mathbf{N}$ represents the internal nonlinear force vector.

2.2. Harmonic response of nonlinear multidegree-of-freedom systems

The method developed by Budak and Özgüven [23] for harmonic response analysis of mdof systems with symmetrical polynomial type nonlinearities later has been generalized [24] for any type of nonlinearity by using describing functions. Nonlinear functions can be represented by (quasi)linear describing functions which have amplitude dependent gain. The basic idea is to apply a sinusoidal input to the nonlinear function and consider the fundamental component of output; hence, describing function is the ratio of the output to the input. Using the method given in Ref. [24], nonlinear differential equations are transformed into nonlinear algebraic equations by using describing functions and representing internal nonlinear forcing vector as a response-dependent matrix multiplied by the displacement vector. The solution technique developed by Özgüven [31] for structural modifications, and successfully employed for the analysis of non-proportionally damped systems [26] and for nonlinear systems [23,24], is used as the solution algorithm. The method suggested decreases the computational time drastically for localized nonlinearities. In this section, forced harmonic response analysis of nonlinear mdof systems using the quasi-linear describing function theory will be briefly summarized.

Consider the mdof nonlinear structure under the effect of harmonic forcing as described by Eq. (2). Then, from this equation one can write

$$\boldsymbol{\alpha}^{-1} \cdot \mathbf{X} + \mathbf{G} = \mathbf{F}, \tag{3}$$

$$\boldsymbol{\alpha} = (\mathbf{K} - \omega^2\mathbf{M} + i\omega\mathbf{C} + i\mathbf{H})^{-1}, \tag{4}$$

where $\boldsymbol{\alpha}$ is the receptance of the linear part of the structure, $\mathbf{X}, \mathbf{F}$ and $\mathbf{G}$ are the amplitude vectors of displacement, external forcing and nonlinear internal forcing, respectively.

Using describing functions it is possible to write $\mathbf{G}$ as a multiplication of displacement-dependent matrix, $\boldsymbol{\Delta}$ and $\mathbf{X}$, where

$$\boldsymbol{\Delta}_{kk} = v_{kk} + \sum_{\substack{j=1 \\ j \neq k}}^{n} v_{kj} \quad \text{and} \quad \boldsymbol{\Delta}_{kj} = -v_{kj}, \tag{5}$$

v_{kj} is the harmonic input describing function and can be described as the equivalent linear complex stiffness for the internal nonlinear force, n_{kj} acting between the kth and jth coordinates, and given by

$$v_{kj} = \frac{i}{\pi||x_k - x_j||} \int_0^{2\pi} n_{kj}\, \mathrm{e}^{-\mathrm{i}\psi}\, \mathrm{d}\psi. \tag{6}$$

By replacing **G** with $\mathbf{\Delta} \cdot \mathbf{X}$, Eq. (2) takes the following form:

$$\mathbf{X} = \mathbf{\Theta} \cdot \mathbf{F}, \tag{7}$$

$$\mathbf{\Theta} = (\mathbf{K} - \omega^2\mathbf{M} + i\omega\mathbf{C} + i\mathbf{H} + \mathbf{\Delta})^{-1}. \tag{8}$$

Here $\mathbf{\Theta}$ is referred to as the pseudo-receptance matrix. It should be noted that $\mathbf{\Delta}$ is a displacement dependent matrix and therefore in calculating $\mathbf{\Theta}$ iteration is to be made. Furthermore, since $\mathbf{\Theta}$ is also a function of displacement, unlike $\boldsymbol{\alpha}$, for each excitation force level a different $\mathbf{\Theta}$ will be obtained. Further details about the method can be found in Ref. [24].

In order to avoid the matrix inversion in Eq. (8), the method suggested by Özgüven [26] for the harmonic vibration analysis of non-proportionally damped structures is used. The method proposed is a structural modification algorithm, and calculates receptance of a modified system from the receptance of the original system (linear system in our case here) and the modification matrix ($\mathbf{\Delta}$ in our case). The application of this method for harmonic vibration analysis of nonlinear systems is given in Ref. [24], and briefly summarized in this paper in Section 2.5.

This general approach of Tanrıkulu et al. [24] for the harmonic vibration analysis of nonlinear mdof systems has successfully been used in several structural dynamics problems, including a recent study of Sanliturk, et al. [18] for the optimization of friction dampers to reduce turbine blade vibrations. In that study, however, harmonic balance method is combined with a different structural modification method using Sherman–Morrison formula, and macro-slip friction model which is not as accurate as micro-slip model is used for friction dampers.

It is important to mention that it is also possible to include higher harmonics into the calculations by following a very similar approach. Although single harmonic represents the system behavior accurately in most cases, there may be cases where higher harmonics are not negligible.

2.3. Two-slope friction model

In modeling dry friction damping mechanisms, macro-slip and micro-slip models are the two theoretical approaches that have been used in previous work. The macro-slip (Fig. 1b) is a widely used approach, in which the entire friction surface is assumed to be either slipping or stuck. Its extensive use is due to its mathematical simplicity and its success in predicting actual responses for low normal load. In the micro-slip approach (Fig. 3), a relatively detailed analysis of stress distribution at the friction interface is carried out. Therefore, the micro-slip model can provide more accurate results, however at the expense of higher computational effort.

The model for the hysteresis curve used in this work is slightly different from the ones described above. The hysteresis model proposed can be considered as an extension of macro-slip model; yet, it also approximately

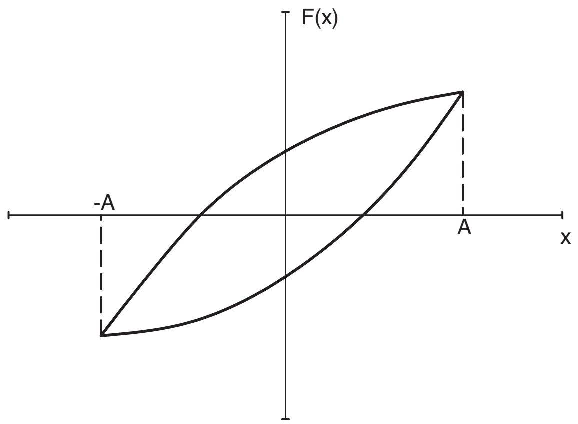

Fig. 3. Hysteresis curve for micro-slip model.

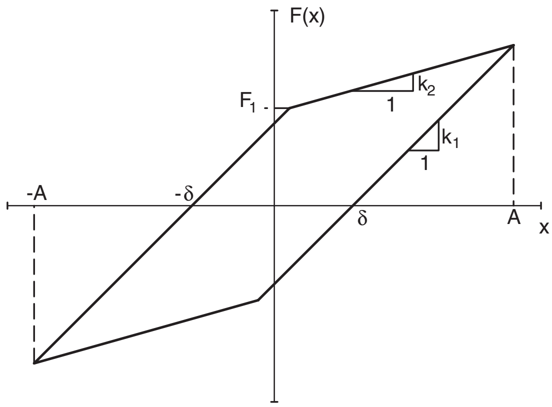

Fig. 4. Two-slope hysteresis curve (left upper corner in the first quadrant).

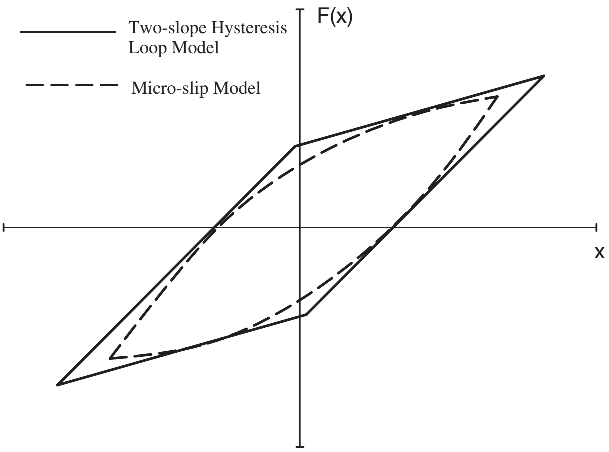

Fig. 5. Comparison of micro-slip and two-slope hysteresis models.

represents the micro-slip behavior (Fig. 4). The hysteresis curve has two slopes instead of one (as it is the case in macro-slip model); however, two slopes make it possible to approximately represent the micro-slip characteristics (Fig. 5). In macro-slip model it is possible to calculate describing function analytically, since the forcing is a combination of linear functions. However, the micro-slip model is expressed in terms of more complicated functions, for which describing functions cannot be evaluated analytically. Therefore, in order to obtain the describing function, numerical integration techniques should be employed, which increases the computational time for mdof systems considerably. On the other hand, the two-slope hysteresis curve proposed here is a combination of linear functions, and therefore, it is possible to obtain analytical expression for the describing function.

2.4. Describing functions for the two-slope friction model

The two-slope friction model proposed is shown in Fig. 4, where A and δ denote the steady-state vibration amplitude and the intersection of the curve with the x-axis, respectively. It should be noted that, the analytical expression for the describing function of such a friction damper depends on the position of the left upper corner of the hysteresis curve, since the order of the integration of the linear functions used to construct the hysteresis curve changes depending on whether the left upper corner is in the first quadrant or in the second quadrant of the F–x plane.

From Eq. (6), the describing function for the hysteresis curve given in Fig. 4, can be determined as [32]

$$v_{\text{real}} = \frac{A(k_1 - k_2) - 2k_1\delta}{A\pi}\sqrt{1 - \left(1 - \frac{2k_1\delta}{A(k_1 - k_2)}\right)^2} + \frac{k_1 - k_2}{\pi}\phi + \frac{1}{2}(k_1 + k_2), \tag{9}$$

$$v_{\text{imaginary}} = \frac{4k_1\delta}{A\pi}\left(1 - \frac{k_1\delta}{A(k_1 - k_2)}\right), \tag{10}$$

where

$$\delta = \frac{(k_1 A - F_1)(k_1 - k_2)}{k_1(k_1 + k_2)}, \tag{11}$$

$$\phi = \arcsin\left(1 - \frac{2k_1\delta}{A(k_1 - k_2)}\right). \tag{12}$$

It should be noted that, $0 \leqslant \phi \leqslant \pi/2$.

Again, from Eq. (6), the describing function for the hysteresis curve, where the left upper corner is in the second quadrant (Fig. 6) can be determined as [32]

$$v_{\text{real}} = \frac{A(k_1 - k_2) - 2k_1\delta}{A\pi}\sqrt{1 - \left(1 - \frac{2k_1\delta}{A(k_1 - k_2)}\right)^2} + \frac{k_1 - k_2}{\pi}\phi - \frac{1}{2}(k_1 - 3k_2), \tag{13}$$

$$v_{\text{imaginary}} = \frac{4k_1\delta}{A\pi}\left(1 - \frac{k_1\delta}{A(k_1 - k_2)}\right), \tag{14}$$

where δ can still be determined from Eq. (11), but ϕ is given by

$$\phi = \pi + \arcsin\left(1 - \frac{2k_1\delta}{A(k_1 - k_2)}\right). \tag{15}$$

It should be noted that, $\pi/2 \leqslant \phi \leqslant \pi$.

2.5. Solution method

In the nonlinear analysis method presented above, a set of n nonlinear ordinary differential equations (Eq. (1)) are converted into a set of n nonlinear complex algebraic equations, Eq. (7). The fundamental harmonic response of the structure to the harmonic external forcing can be obtained by solving Eq. (7)

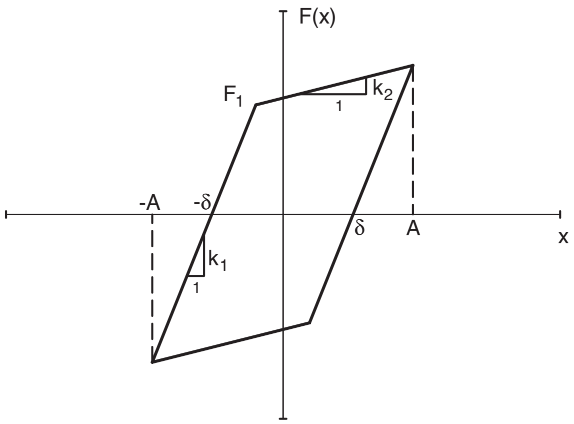

Fig. 6. Two-slope hysteresis curve (left upper corner in the second quadrant).

iteratively for each frequency, ω:

$$\mathbf{X}_{i+1} = \mathbf{\Theta}_i \cdot \mathbf{F}. \tag{16}$$

Here $\mathbf{X}_{i+1}$ denotes the complex displacement amplitude vector at the $(i+1)$th iteration step. The pseudo receptance matrix calculated in the ith step $\mathbf{\Theta}_i$, is determined from Eq. (8), by using $\mathbf{X}_i$ in the computation of $\mathbf{\Delta}$. In order to decrease the number of iterations and obtain a fast convergence, relaxation is applied at every iteration step. The weighted displacements are given by

$$\mathbf{X}^*_{i+1} = \lambda \mathbf{X}_{i+1} + (1-\lambda)\mathbf{X}_i, \tag{17}$$

where λ is a weighting factor having a value between 0 and 2. The values of λ between 0 and 1 are employed to make a non-convergent system converge or hasten convergence by damping out oscillations. The values of λ between 1 and 2 are used to accelerate the convergence of an already convergent system.

The iteration scheme in Eq. (16) requires updating the pseudo-receptance matrix $\mathbf{\Theta}$ at every iteration step. For systems with large numbers of dof, computations of $\mathbf{\Theta}$ through matrix inversion turns out to be expensive. In order to overcome this difficulty, the solution algorithm suggested by Özgüven [26] for the harmonic response analysis of non-proportionally damped structures is employed. The adoption of this method to nonlinear systems is given in Ref. [24], and briefly explained below.

For harmonic excitation, from Eqs. (2), (4) and (6), $\mathbf{X}$ can be obtained as

$$\mathbf{X} = \boldsymbol{\alpha} \cdot (\mathbf{F} - \mathbf{\Delta} \cdot \mathbf{X}). \tag{18}$$

For the pth coordinate, Eq. (18) will take the form

$$\mathbf{X}_p = \sum_{s=1}^{n} \boldsymbol{\alpha}_{ps}\mathbf{F}_s - \sum_{s=1}^{n} \boldsymbol{\alpha}_{ps} \sum_{k=1}^{n} \mathbf{\Delta}_{sk}\mathbf{X}_k. \tag{19}$$

Dividing each term in Eq. (19) by $\mathbf{F}_j$ and setting all other external forces zero, and moreover considering the existence of only the element $\mathbf{\Delta}_{sk}$ of the nonlinearity matrix $\mathbf{\Delta}$ (i.e. all other elements are taken to be zero) it can be written that

$$\mathbf{\Theta}_{pj} = \boldsymbol{\alpha}_{pj} - \boldsymbol{\alpha}_{ps}\mathbf{\Delta}_{sk}\mathbf{\Theta}_{kj}. \tag{20}$$

For $p = k$:

$$\mathbf{\Theta}_{kj} = \frac{\boldsymbol{\alpha}_{kj}}{1 + \boldsymbol{\alpha}_{ks}\mathbf{\Delta}_{sk}} \quad (j = 1, 2, \ldots, n). \tag{21}$$

After calculating $\mathbf{\Theta}_{kj}$ from Eq. (21) for all j, it is possible to determine the remaining $\mathbf{\Theta}_{pj}$ by using Eq. (20) for $p = 1, 2, \ldots, k-1, k+1, \ldots, n$ and $j = 1, 2, \ldots, n$. This process gives the pseudo-receptance matrix when only a single nonlinear term, $\mathbf{\Delta}_{sk}$, is present. If the calculated pseudo-receptance matrix $\mathbf{\Theta}$ is treated as receptance matrix $\boldsymbol{\alpha}$ in Eqs. (20) and (21), a new pseudo-receptance matrix can be calculated by considering another element of $\mathbf{\Delta}$. Thus it is possible to obtain the final pseudo-receptance matrix $\mathbf{\Theta}$ if this process is repeated for all nonzero elements of the nonlinearity matrix $\mathbf{\Delta}$. The algorithm can also be modified to consider all the elements in one column of $\mathbf{\Delta}$ at a time [26], or else, alternatively the matrix inversion formulation presented in the same reference for non-proportionally damped systems can be used by extending the method to nonlinear systems. When the nonlinearity matrix $\mathbf{\Delta}$ is sparse, since no computations are necessary for the zero elements of $\mathbf{\Delta}$, the algorithm given above (or modified form of it) is to be preferred; alternatively, when the nonlinearity is local the matrix formulation may be more efficient. However, either of the algorithms will reduce the computational time drastically. In bladed disk assemblies, since the nonlinear elements are connected to the blades only, the equations of motion are written first for all the blades and then for all the disk elements. As a result, only the $n \times n$ upper left corner of the nonlinearity matrix $\mathbf{\Delta}$, which is a sparse matrix, will be nonzero, and therefore either of the algorithms mentioned above can be used efficiently.

The multiple solutions due to nonlinearity can be obtained by sweeping the frequency in the range of interest from low-to-high and then from high-to-low. The linear response of the system at a starting frequency

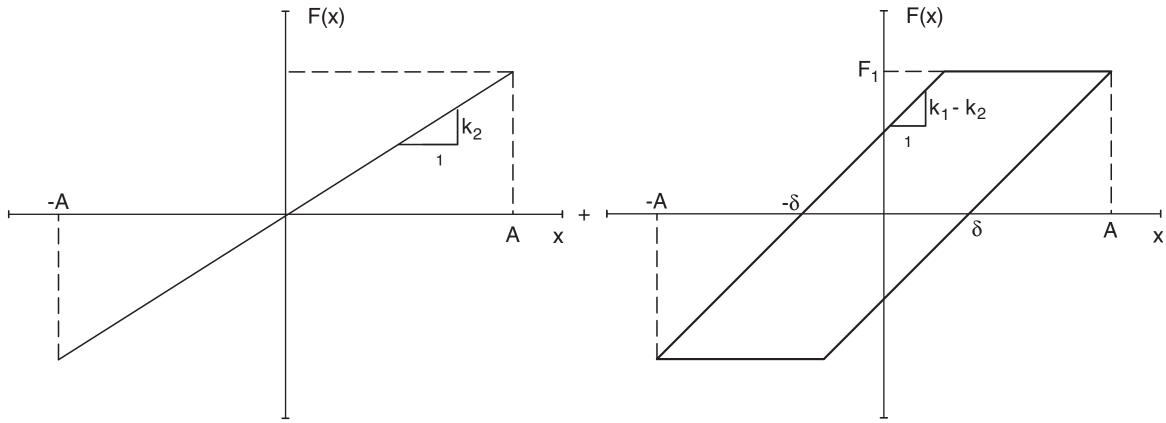

Fig. 7. Decomposition of the proposed two-slope hysteresis curve.

is taken as the initial guess for the displacement vector $\mathbf{X}_1$ at that frequency. However, at other frequencies, the solution obtained at the previous frequency step is taken as the initial guess.

In numerical computations, in order to decrease the computation time, two relaxation parameters are used: one for converging points and one for diverging points. Program switches to any of the supplied relaxation parameters by comparing the error obtained at the present and previous steps, which increases the overall speed of the computations considerably.

For the proposed two-slope hysteresis curve, it is possible to construct the graph as a combination of two functions as shown in Fig. 7. First function shows a force–displacement relationship of a linear spring of stiffness k_2 and the second one is a macro-slip model. In numerical computations, the linear part with slope k_2 is added to the linear part of the equations, and macro-slip model is taken as the only source of nonlinearity. This numerical approach helps to eliminate the convergence problems that may arise in the solution. It should be noted that, describing function for the macro-slip model can be obtained by taking k_2 as zero in Eqs. (9) to (15).

3. Validation of the method

The validation of the method suggested is demonstrated by comparing the results obtained in this study with those given in literature and also with results obtained by time domain simulations. For this reason, a 2-dof system with a friction element connected between the first mass and the ground, and with a harmonic forcing acting on the first mass is analyzed (Fig. 8).

The response of the 2-dof system is studied by keeping the excitation force amplitude constant at 100 N and varying the sliding friction force, F_1, between 50 and 400 N. The values of the parameters used in the calculations for the 2-dof system and the dry friction element are given as

$$m_1 = m_2 = 1\,\text{kg}, \quad c_1 = c_2 = 4\,\text{Ns/m}, \quad k_{11} = k_{22} = 40{,}000\,\text{N/m},$$
$$k_1 = 30{,}000\,\text{N/m} \quad \text{and} \quad k_2 = 100\,\text{N/m}, \quad F_1 = 50, 100, 150, 200, 400\,\text{N}.$$

Fig. 9 shows the comparison of the pseudo-receptances calculated by using the method developed, with those obtained from time domain simulations. When the results obtained by frequency and time domain analyses are compared, it can be seen that, except at some off resonance frequencies for $F_1 = 50$ N, the values calculated by two different solution methods are in excellent agreement. The difference at the off resonance frequencies for this particular case is due the effect of higher harmonics, which are not included in the solution. However, it should be noted that, the results from both methods perfectly agree with each other around the resonance frequency where the maximum vibration amplitude occurs.

The study of the results shows that the resonance frequency of the system increases by increasing sliding friction force, as expected. More interestingly, it is observed that the resonance vibration amplitude increases

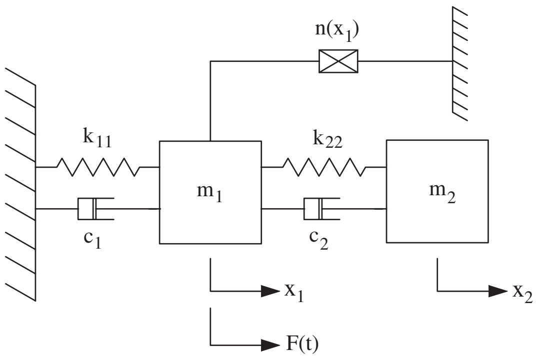

Fig. 8. Two-dof system with dry friction element.

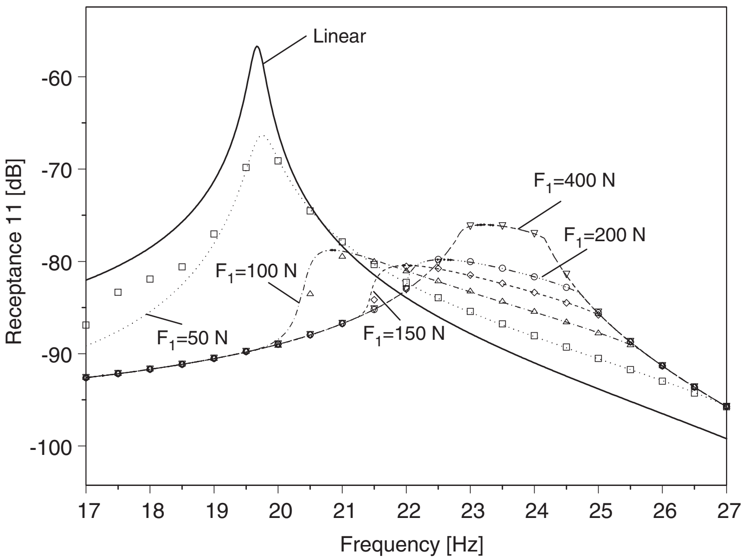

Fig. 9. Pseudo-receptance for different sliding friction force values (discrete points are time simulations).

for $F_1 < 150\,\text{N}$ and $F_1 > 150\,\text{N}$. Therefore, it can be concluded that an optimum value exists for sliding friction force, which is 150 N for this particular case. This observation is very important and it can be used to minimize vibration amplitudes of bladed-disk assemblies by using friction dampers and by tuning the normal load acting on the friction damper. It should be noted that, for the increasing values of sliding friction force, it is hard to obtain slip motion; consequently, system behaves like a linear one in the limiting case (complete stuck), with a stiffness of k_1 added to k_{11}.

The same system given in Fig. 9 is also studied in Ref. [4] with the same system parameter values, but the dry friction damper is modeled by macro-slip, i.e. $k_2 = 0\,\text{N/m}$. The numerical values used for the friction element are

$$k_1 = 30{,}000\,\text{N/m} \quad \text{and} \quad k_2 = 0\,\text{N/m}, \quad F_1 = 50, 75, 100, 125, 150, 200, 300, 400\,\text{N}.$$

The comparison of the solutions obtained by the method developed (Fig. 10) with those given in Ref. [4] shows that both methods give exactly the same results for all sliding friction forces except for $F_1 = 400\,\text{N}$. For this particular case, time domain analysis is performed, and the time simulations are found to be in complete agreement with the frequency domain solutions of the present study.

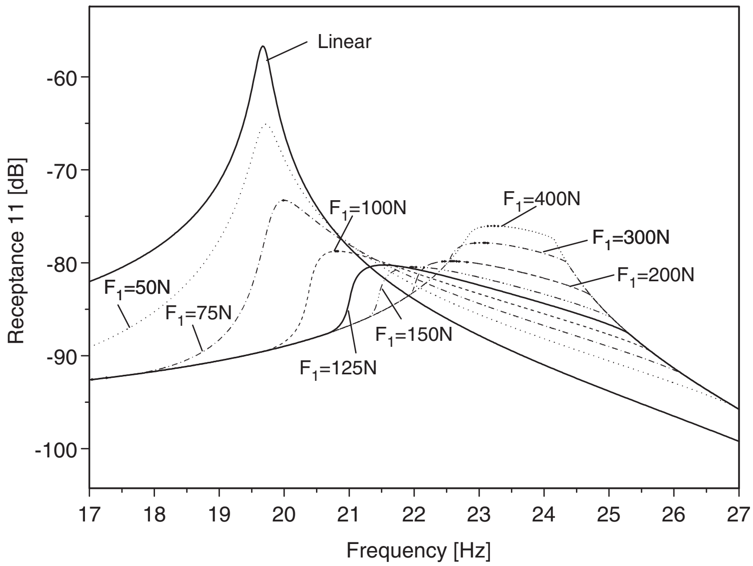

Fig. 10. Pseudo-receptance for different sliding friction force values ($k_2 = 0\,\text{N/m}$).

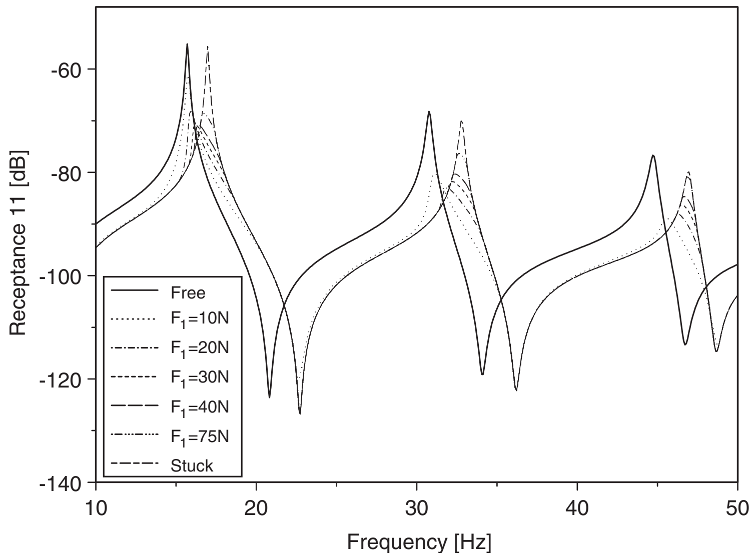

Fig. 11. Pseudo-receptance for tuned BB damper case.

4. Case studies

4.1. Case study 1

A 20-bladed disk system (40-dof) with the following properties is examined:

$$m_j = 0.05\,\text{kg}, \quad M_j = 0.5\,\text{kg}, \quad k_{dj} = 40,000\,\text{N/m}, \quad k_{rj} = 30,000\,\text{N/m}, \quad k_{sj} = 15,000\,\text{N/m},$$
$$\eta = 0.01, \quad k_1 = 10,000\,\text{N/m}, \quad k_2 = 100\,\text{N/m}.$$

Since all system parameters are identical for each disk/blade element in this 40-dof tuned bladed disk assembly, the system has double natural frequencies (eigenvalues). It should be noted that, it is expected to have the first natural frequency as 0, since the system is semi-definite. For identical natural frequencies double

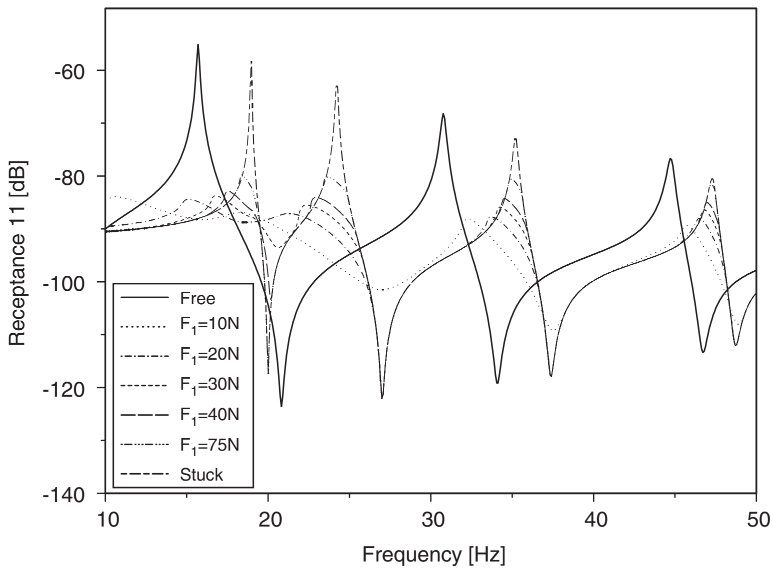

Fig. 12. Pseudo-receptance for tuned BG damper case.

modes, which are similar to each other exist and for these natural frequencies no unique mode shape can be specified. When vibrating freely at the repeated natural frequencies the structure can assume any given combination of the double mode shapes.

The 40-dof bladed disk system is analyzed by adding blade-to-blade (BB) and blade-to-ground (BG) dry friction dampers. A 100 N excitation force is applied on the first blade and for different values of sliding friction force, receptance and pseudo-receptance of the system is given in Figs. 11 and 12 for BB and BG dampers, respectively. When Fig. 11 is inspected, it is seen that the resonance frequency increases with increasing sliding friction force, F_1. When sliding friction force increases, slip can occur at higher relative displacements; therefore, the effect of first slope of the hysteresis loop increases, which also increases the resonance frequency. As the sliding friction force increases, vibration amplitude of the resonance first decreases and then increases; therefore, an optimum value for sliding friction force is present. It is also observed that, this optimum value is different for each resonance, since the relative motion acting on the friction interface is different at each mode. In order to determine the most critical resonance frequencies, Campbell Diagrams, where the natural frequencies of the bladed disk are plotted against the rotational speed, are used. Engine order lines are as well plotted on the same figure and the intersection of these lines with the natural frequencies of the bladed disk system in the operating range gives the critical resonance frequencies. Hence, in order to maximize effective vibration damping, optimization process should be performed at that critical resonance frequency determined from Campbell diagrams. It should further be noted that, if the sliding friction force value is very high, the system becomes completely stuck which shows the effect of shroud; however, vibration amplitudes are still high around the resonances.

When Fig. 12 is analyzed it is observed that the results obtained are similar to the BB damper case: resonance frequency of the system increases as the sliding friction force increases and an optimal value for the sliding friction force exists. On the other hand, since the dampers are placed between the blade and the ground the positive semi-definite system becomes a positive definite system and therefore, the first resonance frequency is not zero. This can be seen from Fig. 12, where the nonlinear system has an additional resonance frequency. The effect of this non-zero resonance frequency is more distinguishable at the first few resonances of the system.

4.2. Case study 2

Since it is hard to obtain identical system parameters due to difficulties in manufacturing and material inhomogeneities; the values of system parameters given in case study 1 are taken as the mean values and 5%

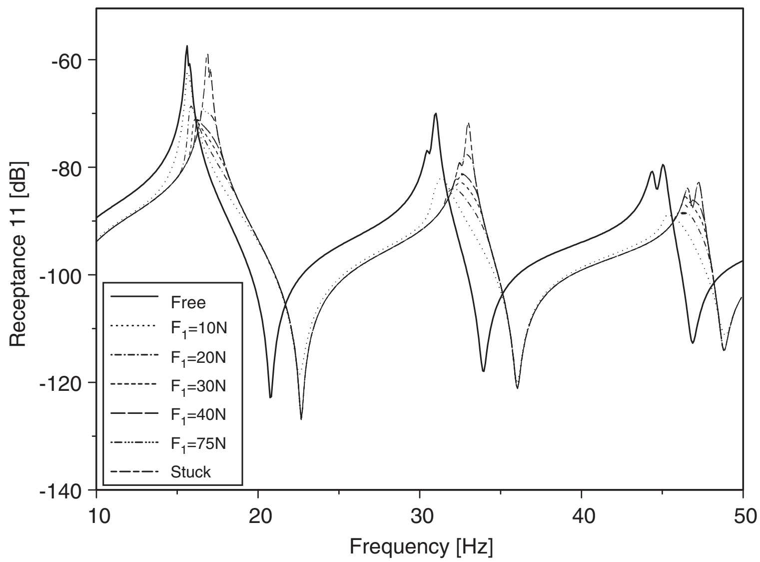

Fig. 13. Pseudo-receptance for mistuned BB damper case.

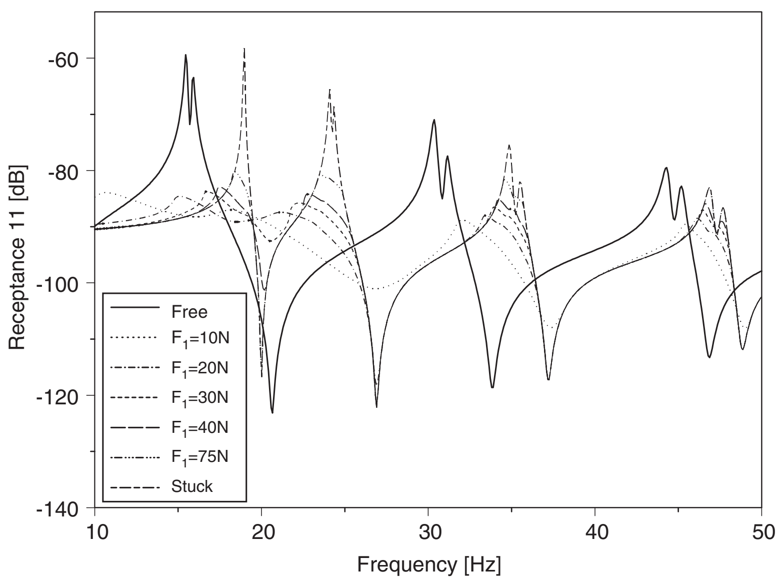

Fig. 14. Pseudo-receptance for mistuned BG damper case.

mistuning is applied (i.e. each parameter used in case study 1 is multiplied by a random number between 0.95 and 1.05, and random numbers are generated by a Gaussian distribution having a mean of 1 and a standard deviation of 0.05). The receptance and pseudo-receptance of the system under the effect of 100 N forcing applied on the first mass is given in Figs. 13 and 14 for BB and BG dampers, respectively. It is observed that, double natural frequencies of the system are separated from each other due to mistuning; however, it should be noted that, even though mistuning has a relatively minor effect on the natural frequencies the effect on mode shapes is significant. For both cases, the resonance frequency of the damped system increases with increasing sliding friction force, F_1, as it is the case in the tuned system. It should be noted that, for BB and BG dampers, tuned and mistuned systems have similar optimal sliding friction force values for each resonance. Moreover, due to damping the double natural frequencies are indistinguishable, and as the damping decreases they become more evident.

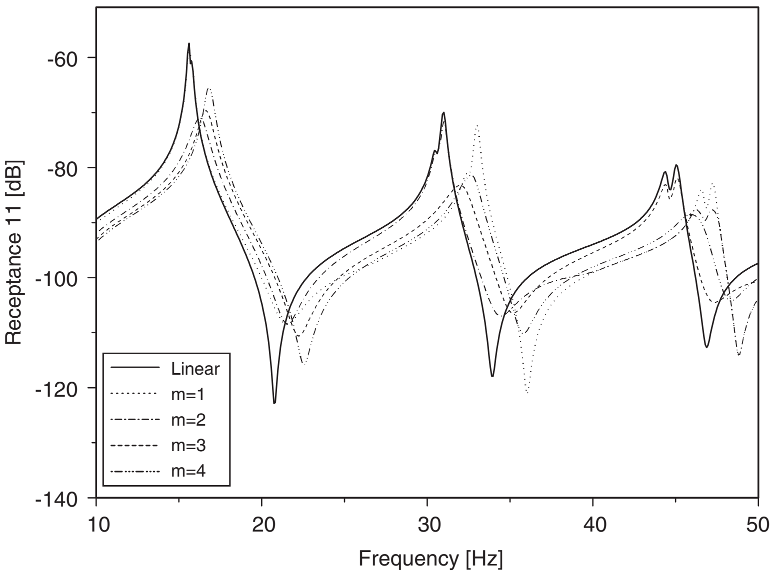

Fig. 15. Pseudo-receptance for engine order excitation: BB damper case.

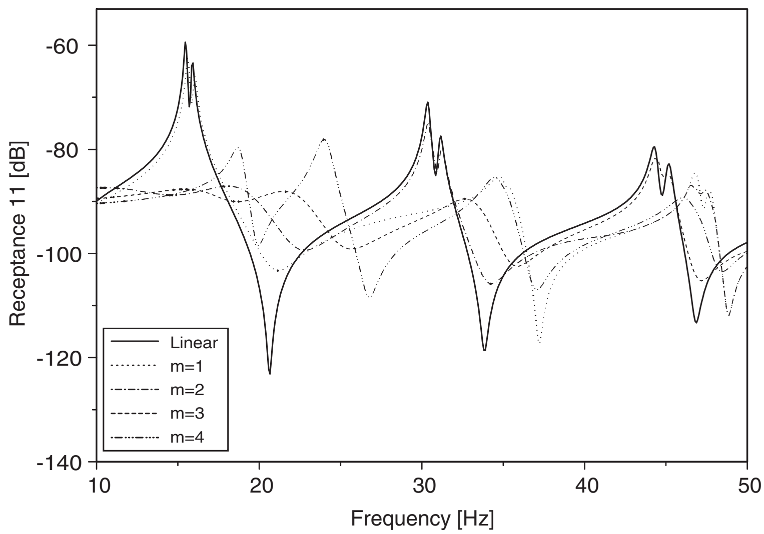

Fig. 16. Pseudo-receptance for engine order excitation: BG damper case.

4.3. Case study 3

In this study, the mistuned system given in case study 2 is analyzed with an engine-order excitation which is defined by

$$\mathbf{f}_j = \mathbf{F}_m \cos\left(m\Omega t + \frac{2\pi m j}{n}\right), \tag{22}$$

where j, m, n, Ω and $\mathbf{F}_m$ are the blade number, engine order, number of blades, rotational speed and excitation amplitude, respectively. Engine-order excitation is generated by the rotation of a bladed disk system past a static pressure or force field; therefore, the strength varies with the angular position of the blade around the disk. Figs. 15 and 16 show the pseudo-receptance versus $m\Omega$ for 1st, 2nd, 3rd and 4th engine orders, where the sliding friction force is 20 N. It should be noted that, the cyclic symmetry of the bladed disk system is destroyed due to the 5% mistuning introduced into the system parameters. Therefore, the resulting motion of the bladed disk system, which is the multiplication of the pseudo-receptance matrix with the excitation force vector, is a combination of all available modes instead of a single mode as in the case of a tuned system [33,34].

5. Conclusion

In this study, a frequency domain method has been developed for forced harmonic response analysis of bladed disks with dry friction dampers. Most of the analysis on bladed disks with dry friction dampers is performed by employing single-dof models, whereas the method used in this work is capable of analyzing mdof systems. The method can be used as an alternative to time domain integration, which is a computationally expensive method for systems with large number of dof. Moreover, the friction model proposed is mathematically simple; yet, it includes microslip effects.

Performed case studies indicate that it is possible to decrease vibration amplitudes by utilizing BB and/or BG dry friction dampers. It is observed that an optimum value for sliding friction force exists, which minimizes the resonance vibration amplitudes; however, it should be noted that, the optimum value of sliding friction force is different for each resonance frequency. Therefore, to determine the optimum value of the sliding friction force, operating and excitation frequencies should be identified in the design stage.

It is observed that, both types of dampers have similar effects in terms of vibration damping; however, low frequency behavior of the system changes if BG dampers are used since the system changes from positive semi-definite to positive definite.

Tuned and mistuned systems are analyzed and quite similar results are obtained for both cases. Frequency splitting is not observed for the mistuned system when the friction damping is effective; on the other hand frequency splitting becomes more apparent as the sliding friction force is increased, since the system converges to the completely stuck case, in which the system is linear without any friction damping.

In the analysis of the nonlinear system single harmonic describing functions are used. It is also possible to include higher harmonics into the calculations by following a very similar approach. Although single harmonic represents the system behavior accurately in most cases, there may be cases where higher harmonics are not negligible.

In this study, the bladed disk system is modeled as a lumped parameter system; on the other hand, it is possible to apply the developed method on a more detailed model such as a finite element model. Since the nonlinearities come from the dampers only, the developed method can be applied effectively and more detailed and specific results can be obtained.

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